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Engineering Notes

Starlight Atmospheric Refraction Model for a Continuous Range of Height

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Introduction

AUTONOMOUS spacecraft navigation, widely known as autonomous orbit determination, is designed to estimate the spacecraft's current position and velocity without having assistance from ground tracking. It conducts the mission in real time using the integrated onboard apparatus. The recent development in measurement techniques and powerful computing has made the autonomous spacecraft navigation feasible [1,2].

Since the advent of the space age, various techniques have been considered for autonomous navigation, such as using magnetometers [3], crosslink measurements [4], Global Positioning System [5], directly sensing horizon using Earth sensor [6], sensing horizon indirectly by starlight atmospheric refraction [7], etc. Developed in the early 1980s, the method of indirect horizon sensing using starlight atmospheric refraction has been proved to be a low-cost, autonomous, and high-accuracy method of celestial navigation. It uses a highly accurate charge-coupled-device star sensor as the only measurement instrument and a mathematical model of starlight refraction in the atmosphere to sense the Earth's horizon, so as to sense the Earth's horizon accurately for high-accuracy determination of spacecraft orbits. In 1980s, the United States initiated the development of MADAN (multimission attitude determination and autonomous navigation), an autonomous spacecraft navigation system using starlight atmospheric refraction to sense the Earth's horizon. This system was tested in space in 1989 and put into operation in the 1990s [8,9]. The major factors affecting the accuracy of celestial navigation by starlight horizon atmospheric refraction are the measurement error of starlight refraction angle and the model uncertainty of starlight atmospheric refraction.

The formulas in [7,8] show that the starlight refraction angle has a direct relationship with the atmospheric density at the refraction tangent height. Uncertainty of atmospheric density model directly leads to the uncertainty of the starlight refraction model, which in turn reduces the accuracy of the navigation system and limits its application. In this Note, using standard atmosphere data in stratosphere expression for variations of temperature, pressure, atmospheric density, and atmospheric density elevation with height is first established. Next, using these models, a starlight atmospheric refraction model with a continuous range of height (from 20 to 50 km) is developed. An empirical expression of starlight atmospheric refraction varying with the tangent height is given as well. The results show that the proposed model is more accurate than the former ones.

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Principle of Autonomous Starlight Refraction Navigation

Light goes straight in homogeneous medium. However, since the atmosphere is inhomogeneous, when light passes through it, the light rays gradually bends due to the refraction in different layers of the atmosphere. Refractive index of light varies with the atmospheric density. As the atmospheric density varies with the height, the refractive index also varies with the height. Therefore, light from the stars is refracted when passing through the atmosphere, which changes the apparent position of stars.

The principle of indirect horizon sensing by starlight atmospheric refraction used in autonomous celestial navigation is explained as follows [7]. Using the star sensor, two stars are observed simultaneously: one unrefracted star for which the starlight passes through the far upper atmosphere (for reference, called the navigational star) and another refracted star for which the starlight grazes the Earth's horizon (called the refractive star). The apparent angle between the navigational star and the refractive star shall be different from the actual value. This difference is called the starlight refraction angle. The variation of refraction angle with atmospheric density (which in turn varies with height) can be modeled accurately. Applying these relationships, the height of the incident starlight ray, which reflects the geometrical position of the spacecraft with respect to Earth, can be obtained accurately (shown in Fig. 1).

Analysis of Several Starlight Atmospheric Refraction Models

In autonomous celestial navigation indirect horizon sensing by starlight atmospheric refraction, several stars must be observed simultaneously or continuously. A former fixed-height starlight atmospheric refraction model is confined in a small range of starlight tangent height (i.e., 25 km) to capture stars, which covers a very small number of available refracted stars and provides low probability for the star sensor to capture the refracted stars. This greatly limits the applications of the method.

In [10], based on the atmospheric density model, an approximate functional expression between starlight refraction tangent height h and starlight refraction angle R is given below (model 1):

$$\begin{cases} R = 0.0221 \cdot \exp(-0.14h) \\ h = -27.23 - 7.143 \ln R \end{cases} \tag{1}$$

Model 1, an empirical measurement equation of autonomous celestial navigation, uses the averaged value of measurements at a height of 25 km. By computing the refraction angle at a starlight tangent height of 25 km in model 1, we can see that the angle is 137.6 arcsec with an error up to 10.4 arcsec compared with real observed data.

In [11], based on theory of atmospheric refraction, stratospheric atmospheric data, atmospheric model, and other factors affecting the starlight refraction measurements, a starlight atmospheric refraction model of continuous range of height was developed and presented here as model 2:

$$\begin{cases} R = 6965.4793 \cdot \exp(-0.15180263h) \\ h = 58.29096 - 6.58750 \ln R \end{cases} \tag{2}$$

At a starlight refraction tangent height of 25 km, the refraction angle computed using model 2 is 156.49 arcsec. The error is 8.5 arcsec, which is smaller than that of model 1, but it is still large when compared with real observed data. Another similar model (model 3) has been given in [12], which is as follows:

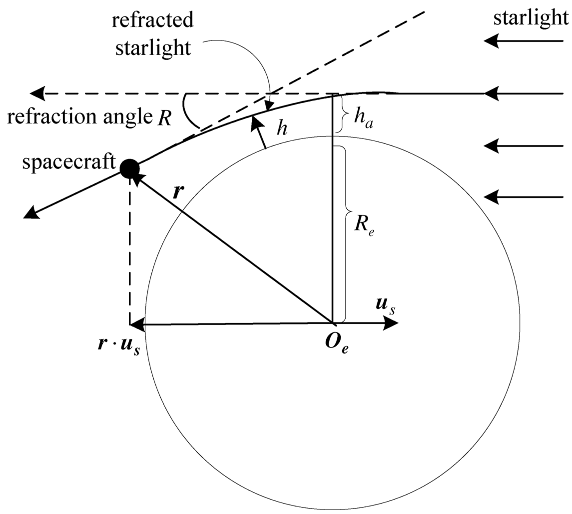

Fig. 1 Spacecraft starlight refraction geometry.

$$\begin{cases} R = 3885.1012 \cdot \exp(-0.1369559h) \\ h = 60.3472 - 7.3016 \ln R \end{cases} \tag{3}$$

Considering that the accuracy of the starlight refraction model directly affects the accuracy of celestial navigation, a novel and accurate starlight atmospheric refraction model for a continuous range of height is developed according to data of U.S. Standard Atmosphere 1976 [13].

Accurate Model of Atmospheric Refraction for a Continuous Range of Height

From the above analysis, it is clear that the starlight refraction angle decreases as the height increases, which implies that the starlight refraction angle is inversely proportional to the distance from the surface of Earth. For the height above 50 km, the refraction is beyond the measurement precision of recent star sensors and will be dealt with as observation noise. For the heights below 20 km, it is hard to detect refracted stars due to the effects of elements in the atmosphere such as vapor and aerosol in stratosphere. Therefore, in order to meet the requirement of indirect horizon sensing by starlight atmospheric refraction, a refraction model for a continuous range of height from 20 to 50 km is set up.

Model of Atmospheric Temperature

The variation of atmospheric temperature with height is very complicated. In the range of height from 20 to 50 km, the temperature is almost independent of the ground, but this variation is not uniform. In the upper air, the temperature gradient is high due to absorption of ultraviolet radiation by ozone. A two-order curve can be used to fit the model of atmospheric temperature variation with the height. Assuming $x = h$ and $y = T$ and applying the curve-fitting algorithm, the following equation can be set up:

$$\begin{bmatrix} m & \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 \\ \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 \\ \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 & \sum_{i=1}^m x_i^4 \end{bmatrix} \begin{bmatrix} c \\ d \\ e \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m y_i \\ \sum_{i=1}^m x_i y_i \\ \sum_{i=1}^m x_i^2 y_i \end{bmatrix} \tag{4}$$

By substituting the data of height and temperature obtained from [13] into Eq. (4), we can get the following solution:

$$\begin{bmatrix} c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 218.920149 \\ -1.015291 \\ 0.044179 \end{bmatrix}$$

Thus, the model of atmospheric temperature variation with height is obtained as follows:

$$T(h) = 218.920149 - 1.015291h + 0.044179h^2 \tag{5}$$

where units of h and T are kilometers and Kelvin, respectively.

Model of Atmospheric Pressure

Atmospheric pressure decreases as height increases. However, its gradient decreases when the height increases. For this phenomenon, the atmospheric pressure variation with height can be modeled as an exponent distribution $P(h) = a \cdot \exp(bh)$. Assuming $x = h$, $y = \ln P$, $A = \ln a$, and $B = b$, according to the algorithm of curve-fitting, the following equation can be written:

$$\begin{bmatrix} m & \sum_{i=1}^m x_i \\ \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m y_i \\ \sum_{i=1}^m x_i y_i \end{bmatrix} \tag{6}$$

Substituting the data of height and pressure (as quoted in [13]) in Eq. (6), the solution is

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 11.414620974 \\ -0.1438481333 \end{bmatrix}$$

Considering the above assumptions, a set of parameters for the distribution is given by

$$\begin{cases} a = e^A = 90637.287961 \\ b = B = -0.143848 \end{cases}$$

Finally, the atmospheric pressure model is obtained as follows:

$$P(h) = 90637.287961 \exp(-0.143848h) \tag{7}$$

where units of h and P are kilometers and pascals, respectively.

Model of Atmospheric Density

Under normal conditions, when the atmospheric temperature is not very low and the pressure is not very high, the interaction of atmospheric molecules and their size can be ignored. The atmosphere can be treated as ideal gas. According to ideal gas law, a functional expression of atmospheric density varying with the height can be given as follows:

$$\rho(h) = \frac{P(h)}{R_0 \cdot T(h)} \tag{8}$$

where R_0 is the gas constant, and $R_0 = 287\ \mathrm{J/(kg.K)}$.

Substituting Eqs. (5) and (7) into Eq. (8), the atmospheric density model within the continuous range of height from 20 to 50 km can be written as

$$\rho(h) = \frac{90637.287961 \exp(-0.143848h)}{287 \cdot (218.920149 - 1.015291h + 0.044179h^2)} \tag{9}$$

The above model, Eq. (9), represents a complicated relationship between atmospheric density and height. As such, its application is also difficult. As can be seen from Eq. (9), atmospheric density varies exponentially from 20 to 50 km; thus, by fitting the exponential curve (based on data of atmospheric density obtained from former models), an empirical formula for variation of atmospheric density with height is shown in Eq. (10):

$$\rho(h) = 1.762162 \cdot \exp(-0.1522204h) \tag{10}$$

where the unit of atmospheric density is $\mathrm{kg/m^3}$, and the unit of height is kilometers.

Model of Atmospheric Density Elevation

For standard atmosphere, the atmospheric density elevation $H_\rho(h)$, the acceleration of gravity, and molecule temperature elevation $T(h)$ vary with height. Based on this, we used a two-order curve, $H_\rho(h) = C + Dh + Eh^2$, to fit the model of atmospheric density elevation. Assuming $x = h$ and $y = H_\rho$ and by applying curve-fitting, the following equation can be derived:

$$\begin{bmatrix} m & \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 \\ \sum_{i=1}^m x_i & \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 \\ \sum_{i=1}^m x_i^2 & \sum_{i=1}^m x_i^3 & \sum_{i=1}^m x_i^4 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^m y_i \\ \sum_{i=1}^m x_i y_i \\ \sum_{i=1}^m x_i^2 y_i \end{bmatrix} \qquad (11)$$

We can calculate $H_\rho(h)$ by using the relation between atmospheric density elevation and atmospheric pressure elevation in [13]. By substituting the pressure, temperature and height data given by [13] and computed atmospheric density elevation into Eq. (11), the solution obtained is as follows:

$$\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 6.518205 \\ -0.037355 \\ 0.001152 \end{bmatrix}$$

Thus, the model of scale atmospheric density is given as follows:

$$H_\rho(h) = 6.518205 - 0.037355h + 0.001152h^2 \qquad (12)$$

where units of both h and H_ρ are kilometers.

Model of Starlight Atmospheric Refraction

All of the factors mentioned above have been modeled accurately based on standard atmosphere data. By substituting Eqs. (7), (10), and (12) into the starlight atmospheric refraction model [6], we get

$$R = k(\lambda)\rho(h)\sqrt{\frac{2\pi(R_e + h)}{H_\rho(h)}} \qquad (13)$$

where R_e is radius of the Earth, and $k(\lambda)$ is the dispersion parameter for air at wavelength λ. By supposing that λ is 0.7 μm, then $k(\lambda) = 2.2517 \times 10^{-7}$.

The model of starlight refraction angle varying with the starlight refraction tangent height within the continuous range of height from 20 to 50 km is obtained as follows:

$$R = 2.25 \times 10^{-7} \cdot 1.762162 \cdot \exp(0.152204h) \cdot \left(\frac{2\pi(R_e + h)}{6.518205 - 0.037355h + 0.001152h^2}\right)^{\frac{1}{2}} \qquad (14)$$

In the method of autonomous navigation using the refracted starlight to sense the Earth's horizon, the starlight refraction tangent height shall be calculated by the relationship with the starlight refraction angle. Therefore, for better applications, empirical formulas of starlight atmospheric refraction model using the data of refraction angle are obtained from Eq. (14) as

$$\begin{cases} R = 7056.436417 \cdot \exp(-0.155248h) \\ h = 57.081067 - 6.441326 \ln R \end{cases} \qquad (15)$$

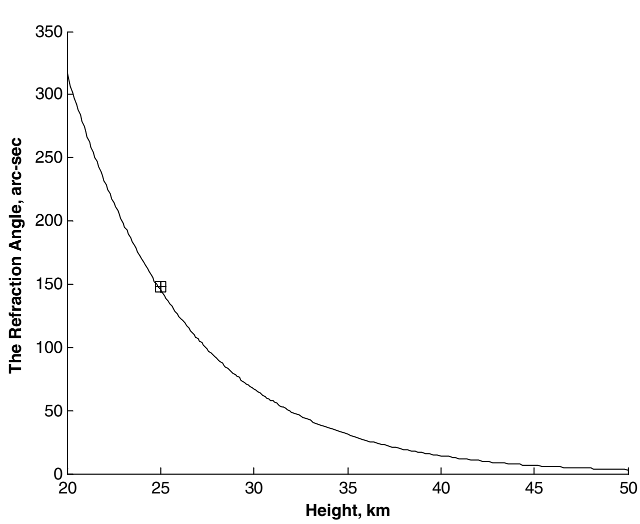

Fig. 2 Variation of starlight atmospheric refraction angle with height.

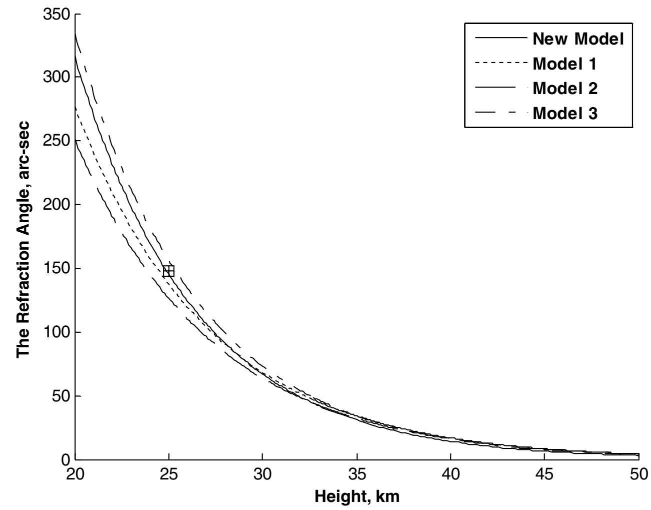

Fig. 3 Comparison of accuracy between new model and old ones.

where units of R and h are arc seconds and kilometers, respectively. The relationship between starlight refraction angle R and starlight refraction tangent height h is illustrated in Fig. 2.

The starlight refraction angle at tangent height of 25 km calculated by the new model is 147.8 arcsec, which has an error of 0.2 arcsec compared with the observed data. Thus, the new model provides far more accurate measurements than those of others, as given in [10–12]. A comparison of models is shown in Fig. 3.

Conclusions

The accuracy of autonomous spacecraft navigation is primarily limited by errors of starlight refraction angle computed using starlight atmospheric refraction model. The new starlight atmospheric refraction model, given by empirical formula within the continuous range of height from 20 to 50 km in the stratosphere, has taken into account all the factors affecting starlight atmospheric refraction, including atmospheric density, temperature, pressure, and density elevation. The model proposed in this Note represents the change of starlight atmospheric refraction better than the previous models. It offers improved applications for autonomous celestial navigation. Using this new model, the starlight refraction navigation system can realize the positioning as long as the star sensor tracks refracted light rays originated from stars at any tangent height between 20 and 50 km. It covers more available numbers or a high probability of tracked stars, thus greatly improving the accuracy, availability, and applicability of autonomous celestial navigation systems using the starlight refraction.

The high-accuracy autonomous spacecraft navigation is made possible by increasing the accuracy of orbit determination based on measurements of the starlight refraction angle through optimal estimation and improved filter algorithms.

Acknowledgments

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